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Application of Mathematical Analysis to the Study of the Influence of Physiological and Ecological Factors on the Change of Fat Contents of the Adult Sprat in the Bay of Gdańsk

by

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In our paper we make an attempt to fit mathematical formulae, mainly taken from the multifarious correlation calculus, to the existing dependence between the contents of fat in the sprat of the Bay of Gdańsk and physiological (gametogenesis) as well as environmental factors (water temperature and abundance of plankton).

We shall base our investigation on the content of fat completed in the years 1954-62. During that period we have examined a total of 211 samples from sprat catches taken in the Bay and in the Deep of Gdańsk.

The changes in contents of fat in particular months of a year's cycle established by analysis are shown in Diagram 1, which shows that the year's cycle of changes of fat in the adult sprat of the Bay of Gdańsk is divided into distinct phases:-

1) a decreasing phase, corresponding to the winter/spring months, and

2) an accumulating phase, corresponding to the summer/autumn period.

We shall hereafter analyse these two phases separately.

The Decreasing Process of Fat Content

The decreasing process of fat content in the sprat of the Bay of Gdańsk commences already in December. Its starting point is the maximum provision of October and November fat accumulated in the organism as a result of the summer/autumn feeding (3).

The decrease in fat content lasts till May/June. The decreasing spend of fat quantity from the starting level - in October/November - is not the same during the next winter and spring months. The lowest takes place in December, January, February, then it decreases several times in March, April, May, after which it stabilises in May/June on the lowest level for the year. Comparing this with the cycle of growth of the seminal glands we may affirm, that the phase of low decrease of fat takes place during the period of the small cycle of gametogenesis. The noted stabilisation in May/June at the lowest level is constant during the months of full spawning, when the sprat ejaculating by parts its seminal products, at the same time, however, in the intervals, it already feeds intensively.

Supposing that during each cycle of gametogenesis the decrease of fat for each unit of time (T) takes place at the same rate, we may write down:-

$$\frac{dF}{dt} = -r$$

where F means the quantity of fat in the sprat and r is the coefficient of reduction of fat.

Solving that equation we finally get the following formula for the quantity of fat in time (T):-

$$F_t = F_o e^{-rt}$$

where F_o is the reserve of fat in the beginning,
F_t = the quantity of fat after a time t,
e = the base of natural logarithms.

From the equation (1) we obtain the instantaneous coefficient r:-

$$r = \frac{\log_e F_o - \log_e F_t}{t}$$

The coefficients of fat reduction for each of the investigated years 1952-1962 have been obtained from Formula (2) separately for both the cycles of gametogenesis. We have given their values in Table 1. If from the data shown in that table we exclude the extreme values, then we can see that the remaining coefficients generally change in a small range and do not much differ from the arithmetical mean of overall years.

The causes of the great oscillations of the coefficients of fat reduction from the average of several years are unknown to us, may be they are caused by a series of factors existing in the environment as well as in the organism of the sprat. If we investigate the speed of fat decrease in the small and large cycle of gametogenesis, we shall see that some evident dependences appear hear. To each diminuation of speed of the fat decrease in the small cycle corresponds, as a rule, an augmentation of speed of that process during the next phase of the great cycle of gametogenesis (viz.: coefficients for 1954/55 and 1956/57). This rule apparently holds true for the inverse also:- each acceleration of the fat decrease at the initial phase of the gametogenesis causes its slowing down during the next phase (viz.: coefficients for 1957/58 and 1959/60). Undoubtedly, the speed of fat transformation in both cycles, as in general the speed of transformation of matter in the sprat maturing for spawning, depends on the environmental conditions, though we have not been able to state this interdependence in an irrefutable way after having analysed the eventual influence of some factors as for example the influence of temperature of water.

Admitting that in Table 1 the averages for several years represent correctly enough the speed of fat decrease in the most frequent mean surrounding conditions then the equation (1) for each cycle of the gametogenesis will get the following form:-

A) the small cycle of gametogenesis

$$F_t = F_o e^{-0,0041\ t} \qquad (3)$$

where F_o is the mean quantity of fat in October/November

B) the great cycle of gametogenesis

$$F_t = F_o e^{-0,0117\ t} \qquad (4)$$

where F_o = quantity of fat at the end of Febr.

Process of Fat Accumulation

The process of fat accumulation in the sprat of the Bay of Gdańsk lasts from June/July to October/November. In the last two months the quantity of fat in the sprat reaches its yearly maximum and at the same time remains more or less on the same level.

From the investigations laid out by W. Mańkowski (4) and by us (1) it results that the fat content mainly depends on ecological factors (thermics of water and quantity of plankton). In Table 2 we have given the quantity of fat confirmed by us as compared with the temperature of water during the feeding months of the sprat (July/September) and in the winter months, and also compared with the abundance of plankton noted by W. Mańkowski according to the 5 degree scale.

The data presented in that table have served to estimate the correlative coefficient (We cite them in Table 3).

From the comparison of the correlative coefficients we can draw the following conclusions:-

1. The quantity of fat accumulated by the sprat in autumn mainly depends on thermic factors; the food stock abundance plays an important part; however, as it seems, it is a rather secondary role.

2. The decisive factor as to the quantity of plankton product as well as to the quantity of accumulated fat in the sprat is the reserve of heat energy left at the end of winter in the middle layers of water depth.

Applying the data given in Tables 2 and 3 we obtained the coefficient of partial regression for factors X_1, X_2, X_3 to the quantity of fat (Y). These coefficients have the following values:-

X_1	(temperature of water in summer)	to Y (content of fat)	=	o,48o6
X_2	(temperature of water in winter)	to Y (content of fat)	=	o,6146
X_3	(quantity of plankton)	to Y (content of fat)	=	o,o647

The above-cited coefficients of partial regression have served to form the following equation of the multivarious regression type.

$$Y = A + b_1X_1 + b_2X_2 + \ldots\ldots\ldots\ldots b..X..$$

In our case the euqation has taken the following aspect:-

$$Y = -4,84 + 1,o9\ X_1 + 1,31\ X_2 - o,13\ X_3 \qquad (5)$$

By use of the equation (5) we have estimated the contents of fat in October and November for the investigated years 1954-61. The values have been compared in Table 5 with the values observed by us, where for each year we have estimated the respective deviations. These deviations do not, in general, exceed 1%, the only exception being the year 196o. In that year probably the quantity of fat appeared through the action of some extra factor beside those already taken in the estimation, i.e. the temperature of water and quantity of plankton.

The small range of deviations of the fat contents estimated on the base of the observed contents proves that equation (6) in a satisfactory way expresses the relation between the fat accumulation process and the biotic and abiotic factors of the environment, which we have considered in the evaluation.

Conclusions

1) The change of fat content in the sprat of the Bay of Gdańsk in a year's cycle may be represented by mathematical equations in a satisfactory way. In the winter/spring period of decrease of fat, equations taken from the differential and integral calculus fit best (equations 1-4). In the summer/autumn period of regeneration of fat reserve by the sprat, which mainly depends on the balance of heat of water and the production of plankton, the abundance of that reserve may be estimated by the equation of the multivarious regression (equation 5).

2) The rate of the winter/spring fat decrease mainly depends on the rapidity of sexual maturing of the sprat. During the great cycle of gametogenesis the rate of fat decrease on the average is three times quicker than the preceding small cycle.

3) From the correlation analysis it results that the reserve of heat energy remaining in the middle layers of water depth after the period of winter cooling is decisive for the quantity of production of plankton and consequently for the quantity of fat accumulated by the feeding sprat during the autumn months.

4) The equations (3 - 4 - 5) may be useful to the forecasting of fat quantity in the sprat of the Bay of Gdańsk. The very high dependence between the temperature of middle layers in the last months of winter and the quantity of fat in the sprat in autumn permits to foresee the latter quantity with a fairly good accuracy already in February/March each year.

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Table 1. Instantaneous coefficients of fat reduction calculated for the winter/spring period of 1954-1962

Cycle of gametogenesis		small cycle	great cycle		
Months of cycle duration		XII - I - II	III - IV - V		
Duration of cycle (in days)		9o	9o		
Time adopted for:	F_o	average fat content in X-XI	fat content at the end of February		
	t_o	31.XI	28.II		
Years of observation	1954/55	(o.oo28)	Years of observation	1955	o.o154
	1955/56	o.oo46		1956	o.o139
	1956/57	o.oo11		1957	o.o148
	1957/58	o.oo49		1958	o.o1o9
	1958/59	o.oo46		1959	o.oo84
	1959/6o	o.oo52		196o	o.oo96
	196o/61	o.oo42		1961	o.o112
	1961/62	o.oo42		1962	o.oo99
A v e r a g e		o.oo41	A v e r a g e		o.o117

Table 2. Quantity of fat in the sprat of the Bay of Gdańsk in October and November and temperature of water during the feeding period, the thermic character of winter and the abundance of zooplankton.

Years of observation	Mean content of fat in October and November	Mean temperature of water at the surface layer near the Hel Peninsula in July and September	Mean temperature in the Deep of Gdańsk in the 3o-5o m layer in February/March	Quantity of plankton in summer acc. to Mańkowski
	Y	X_1	X_2	X_3
	%	°C	°C	
1954	13.7	16.5	1.3	3
1955	19.1	18.7	2.6	4
1956	11.8	15.8	-o.2	1
1957	16,8	16.9	2,7	5
1958	14.9	17.2	1.1	3
1959	17.5	18.8	2.4	3
196o	16.3	17.o	1.o	2
1961	17.2	17.o	2.9	3

Table 3. Relations between the temperature of water in summer (X_1) and winter (X_2) between the quantity of plankton (X_3) and the quantity of fat in the autumn sprat (Y)

Relations:	Temperature of water in summer	Temperature of water in winter	Quantity of zooplankton
	X_1	X_2	X_3
	Coefficient of correlations		
Relations between the quantity of accumulated fat in autumn and X_1, X_2, X_3	o.845o	o.8696	o.6587
Relations between the quantity of plankton and X_1 and X_2	o.4572	o.8195	-
Relation between the temperature of water in summer and X_2	o.641o	-	-

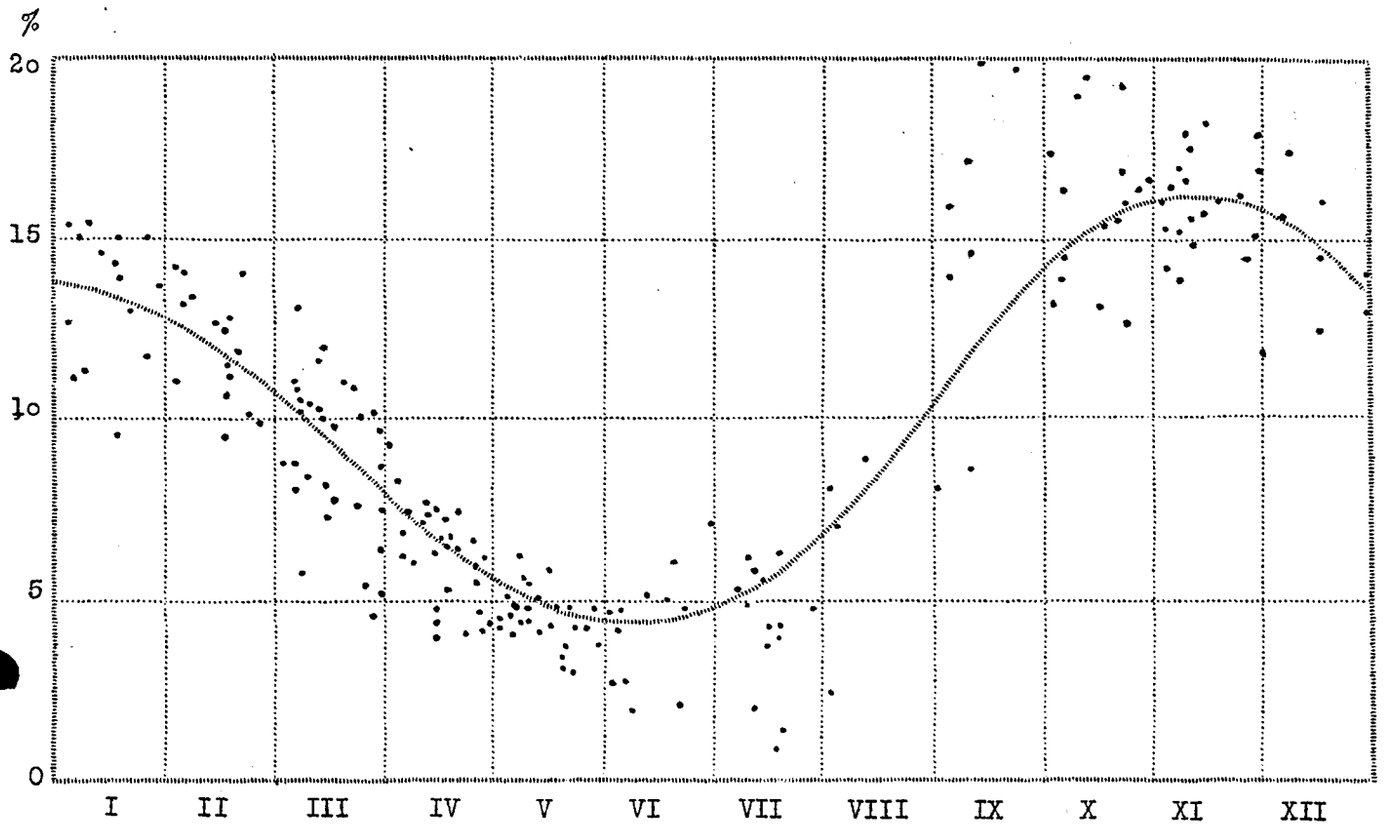

Figure 1. Oscillations in a year's cycle of the quantity of fat in the sprat of the Bay of Gdańsk during the years 1954-1962.